

FEDERAL PUBLIC SERVICE COMMISSION



COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2011

<u>Roll Number</u>

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100
NOTE: (i) Attempt FIVE questions in all by selecting THREE questions from SECTION – A and TWO questions from SECTION – B . All questions carry equal marks. (ii) Use of Scientific Calculator is allowed. (iii) Extra attempt of any question or any part of the attempted question will not be considered.	

SECTION - A

- Q.1.** (a) Prove that every non-empty set of real numbers that has an upper bound also has an supremum in \mathbb{R} . (10)
- (b) If $x \in \mathbb{R}$, set of real numbers, then there exists $n \in \mathbb{N}$ such that $x < n$. (10)
- Q.2.** (a) Define continuity of a function at a point and also prove that if f and g be functions on A to \mathbb{R} , where $A \subseteq \mathbb{R}$ then $f + g$ and $f \cdot g$ are continuous at C . (10)
- (b) If $f : I \rightarrow \mathbb{R}$ is differentiable at $C \in I$, then f is continuous at C . (10)
- Q.3.** (a) Evaluate $\int_1^5 \frac{dx}{\sqrt[3]{x-2}}$. (08)
- (b) (i) Define Complete metric space. (04)
- (ii) Prove that a sequence of real numbers is convergent iff it is a Cauchy sequence. This theorem is not in metric space, for justification give one example. (08)
- Q.4.** (a) Let (X, d) be a metric space and A a subset of X . Then prove that
- (i) Interior A° of A is an open subset of X . (05)
- (ii) A° is the largest subset of X contained in A . (05)
- (b) State and prove Mean value theorem. (10)
- Q.5.** (a) If $\sum a_n$ converges absolutely then $\sum a_n$ converges. (10)
- (b) Find the area enclosed by the parabola $y^2 + 16x - 71 = 0$ and the line $4x + y + 7 = 0$ (10)

SECTION – B

- Q.6.** (a) Let $Z = (\cos \theta + i \sin \theta)$. Then prove that $Z^n = \cos n\theta + i \sin n\theta$ for all n . (10)
- (b) Using De Moivre's Theorem evaluate $\left(\frac{\sqrt{3} - i}{\sqrt{3} + i} \right)^6$. (10)
- Q.7.** (a) Expand $f(x) = x^2$, $0 < x < 2\pi$ in a Fourier series if period is 2π . (10)
- (b) If $f(z)$ is analytic inside a circle C with centre at a , then for all Z inside C (10)
- $$f(z) = f(a) + f'(a)(z-a) + \frac{f''(a)}{2!}(z-a)^2 + \dots$$
- Q.8.** (a) Evaluate the integral by using Cauchy integral Formula (10)
- $$\int_C \frac{(4-3z)dz}{z(z-1)(z-2)} \quad \text{where } C \text{ is a circle } |z| = \frac{3}{2}.$$
- (b) Prove that $\int_0^{2\pi} \frac{d\theta}{1-2p\cos\theta-p^2} = \frac{2\pi}{1-p^2}$. (10)
