NOTE:  
(i) Attempt FIVE questions in all by selecting at least THREE questions from SECTION–A and TWO questions from SECTION–B. All questions carry EQUAL marks.
(ii) Use of Scientific Calculator is allowed.

SECTION – A

Q.1. (a) If \( f \) is continuous on \([a,b]\) and if \( \infty \) is of bounded variation on \([a,b]\), then \( f \in R(\infty) \) on \([a, b]\) i.e. \( f \) is Riemann – integrable with respect to \( \infty \) on \([a,b]\)  
(10)

(b) Let \( \sum a_n \) be an absolutely convergent series having sum \( S \). then every rearrangement of \( \sum a_n \) also converges absolutely & has sum \( S \).  
(10)

Q.2. (a) For what +ve value of \( P \), \( \int_{-1}^{0} \frac{dn}{(1-x)^P} \) is convergent?  
(10)

(b) Evaluate \( \int_{1}^{5} \frac{dx}{\sqrt{x-2}} \)  
(10)

Q.3. (a) Find the vertical and horizontal asymptotes of the graph of function:  
\( f(x) = (2x + 3) \sqrt{x^2 - 2x + 3} \)  
(10)

(b) Let  
(i) \( y = f(x) = \frac{(x + 2)(x-1)}{(x-3)^2} \)  
(ii) \( y = f(x) = \frac{(x-1)}{(x+3)(x-2)} \)  
(10)

Examine what happens to \( y \) when \( x \to -\infty \) & \( x \to +\infty \)

Q.4. (a) Find a power series about 0 that represent \( \frac{x}{1-x^3} \)  
(6)

(b) Let \( \sum s \) be any series, Justify.  
(5+5+4)

(i) if \( \lim_{n \to \infty} \left| \frac{S_{n+1}}{S_n} \right| = r < 1 \), then \( \sum s \) is absolutely convergent.

(ii) if \( \lim_{n \to \infty} \left| \frac{S_{n+1}}{S_n} \right| = r \) and \( (r > 1 \text{ or } r=\infty) \), then \( S \) diverges.

(iii) if \( \lim_{n \to \infty} \left| \frac{S_{n+1}}{S_n} \right| = 1 \), then we can draw no conclusion about the convergence or divergence.
Q.5. (a) Show that \[ \int_0^\infty \sin^{2n-1} \theta \cos^{2n-1} \theta \, d\theta = \frac{\Gamma(m) \Gamma(n)}{2\Gamma(m+n)}; m, n > 0 \] (10)

(b) Prove that \[ \beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}; m, n > 0 \] (10)

Q.6. (a) Let A be a sequentially compact subset of a matrix space X. Prove that A is totally bounded. (10)

(b) Let A be compact subset of a metric space (X,d) and let B be a closed subset of X such that \( A \cap B = \emptyset \) show that \( d(A,B) > 0 \) (10)

SECTION – B

Q.7. (a) Show that if \( \tan \theta \) is expanded into Laurent series about \( \theta = \frac{\pi}{2} \), then

(i) Principal is \( \frac{-1}{z - \frac{\pi}{2}} \)

(ii) Series converges for \( 0 < |z - \frac{\pi}{2}| < \frac{\pi}{2} \)

(b) Evaluate \( \int_{C} \frac{e^{zt}}{z^2(z^2 + 2z + 2)} \, dz \) around the circle with equation \( |z|=3 \). (10)

Q.8. (a) Expand \( f(x) = x^2 \); \( 0 < x < 2 \pi \) in a Fourier series if period is \( 2 \pi \). (10)

(b) Show that \( \int_0^\pi \cos x \, dx = \frac{\pi}{2} e^{-x}; x \geq 0 \) (10)

Q.9. (a) Let \( f(z) \) be analytic inside and on the simple close curve except at a pole of order \( m \) inside \( C \). Prove that the residue of \( f(z) \) at \( a \) is given by \( a_{-1} = \lim_{z \to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \{(z-a)^m f(z)\} \) (10)

(b) If \( f(z) \) is analytic inside a circle \( C \) with center at \( a \), then for all \( Z \) inside \( C \).

\[ f(z) = f(a) + f'(a)(z-a) + f''(a)\frac{(z-a)^2}{2!} + f'''(a)\frac{(z-a)^3}{3!} + ... \] (10)

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