



**FEDERAL PUBLIC SERVICE COMMISSION**  
**COMPETITIVE EXAMINATION FOR**  
**RECRUITMENT TO POSTS IN BS-17**  
**UNDER THE FEDERAL GOVERNMENT, 2015**

Roll Number

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**PURE MATHEMATICS, PAPER-I**

**TIME ALLOWED: THREE HOURS**

**MAXIMUM MARKS = 100**

- NOTE:** (i) Attempt **ONLY FIVE** questions in all, by selecting **THREE** questions from **SECTION-I** and **TWO** questions from **SECTION-II**. **ALL** questions carry **EQUAL** marks.
- (ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
- (iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- (iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- (v) Extra attempt of any question or any part of the attempted question will not be considered.
- (vi) **Use of Calculator is allowed.**

SECTION-I

- Q.No.1. (a) Let  $H$  be a subgroup of a group  $G$ . Prove that the normalizer of  $H$  in  $G$  (i.e.  $N_G(H)$ ) is a subgroup of  $G$ . 10
- (b) Prove that a group of prime order is cyclic. 10
- Q.No.2. (a) Write three non-isomorphic groups of order 12. 10
- (b) Prove that a group  $G$  is isomorphic to a subgroup of group of automorphisms of  $G$ . 10
- Q.No.3. (a) Construct Cayley's table for Multiplication Modulo 7 of  $Z_7 - \{0\} = \{\bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}\}$ . 8  
(4+3+1)  
Show that  $Z_7$  is an integral domain. (You may use Cayley's table.)  
Is  $Z_7$  a field? Justify your answer.
- (b) Give an example of zero divisor in  $Z_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$ . 5  
(2+1+2)  
Is  $Z_6$  an integral domain? Justify your answer.
- (c) What is FIELD EXTENSION? 7  
Verify that the field  $\mathbb{Q}[\sqrt{5}] = \{x + y\sqrt{5} : x, y \in \mathbb{Q}\}$  is an extension of  $\mathbb{Q}$ .
- Q.No.4. (a) Show that  $W = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$  is a subspace of the vector space  $\mathcal{M}_2(\mathbb{R})$  consisting of all  $2 \times 2$  matrices over  $\mathbb{R}$ . 10
- (b) Prove that if a subset  $\{v_1, v_2, \dots, v_k\}$  of a vector space  $V$  is linearly dependent then one vector among  $v_1, v_2, \dots, v_k$  is linear combination of the remaining vectors. 4
- (c) (1) What is dimension of  $\mathbb{R}^3$ . 6  
(2) Write a basis of  $\mathbb{R}^3$ . (1+1+2+2)  
(3) Is  $\{(1, 1, 0), (1, 1, 2), (1, 0, 1), (0, 1, 2)\} \subseteq \mathbb{R}^3$  linearly dependent or independent? Justify your answer.  
(4) Is  $\{(0, 0, 0), (1, 1, 2), (1, 0, 1)\} \subseteq \mathbb{R}^3$  linearly dependent or independent? Justify your answer.

## PURE MATHEMATICS, PAPER-I

- Q.No.5. (a) Define eigen value of a square matrix. 10  
Find eigen values and eigen vectors of  $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- (b) Find reduced echelon form of the matrix 10  
$$A = \begin{bmatrix} 4 & 3 & 7 \\ 1 & 1 & 5 \\ 4 & 5 & 7 \end{bmatrix}$$
- (c) Let  $\lambda_1, \lambda_2, \dots, \lambda_n$  be eigen values of a square matrix  $A = [a_{ij}]$ . What are  $|A|$  and  $\text{trace}(A)$  in terms of  $\lambda_i$ 's?

### SECTION-II

- Q.No.6. (a) Find equations of tangent plane and normal line at a point  $(x_1, y_1, z_1)$  of ellipsoid  $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} = 1$  10
- (b) Find equation of the ellipse centered at the origin, a focus at (3, 0) and vertex at (3, 0). 5
- (c) Find the polar equation of a parabola  $x = 8y^2$ . 5
- Q.No.7. (a) Find the equation of elliptic paraboloid  $x = y^2 + z^2$  In spherical coordinates. 10
- (b) Convert the following equation of quadratic surface to standard form. What is this surface?  
 $4x^2 + y^2 + 4z^2 - 16x - 2y + 17 = 4$
- Q.No.8. (a) Find curvature of the space curve  $\vec{r}(t) = 2t\hat{i} + t^2\hat{j} + \frac{1}{3}t^3\hat{k}$  10
- (b) (1) Find first fundamental form of the surface  $\vec{r}(u, v) = (\cos u, \sin u, v)$  10  
(6+4)
- (2) Write formulae for normal and Gaussian curvature of a surface  
 $\vec{r} = \vec{r}(u, v)$

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