FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR

RECRUITMENT TO POSTS IN BPS-17 UNDER

THE FEDERAL GOVERNMENT, 2010

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: 3 HOURS

MAXIMUM MARKS: 100

NOTE:
(i) Attempt FIVE questions in all by selecting at least THREE questions from SECTION–A and TWO questions from SECTION–B. All questions carry EQUAL marks.
(ii) Use of Scientific Calculator is allowed.

SECTION – A

Q.1. (a) Let W be a subspace of a finite dimensional vector space V, then W is finite dimensional and \( \dim (W) \leq \dim (V) \). Also if \( \dim (W) = \dim (V) \), then \( V = W \). \( \textbf{(10)} \)

(b) Let \( V & W \) be vector space and let \( T : V \rightarrow W \) be a linear if \( V \) is finite dimensional, then \( \text{nullity} (T) + \text{rank} (T) = \dim V \). \( \textbf{(10)} \)

Q.2. (a) Show that there exist a homomorphism from \( S_n \) onto the multiplication group \{ -1, 1 \} of 2 elements \( (n \geq 1) \). \( \textbf{(7)} \)

(b) If \( H \) is the only subgroup of a given finite order in a group \( G \). Prove that \( H \) is normal in \( G \). \( \textbf{(7)} \)

(c) Show that a field \( K \) has only two ideals (namely \( K \) & \( 0 \)). \( \textbf{(6)} \)

Q.3. (a) Find all possible jordan canonical forms for 3x3 matrix whose eigenvalues are \(-2, 3, 3\) \( \textbf{(10)} \)

(b) Show that matrix
\[
\begin{pmatrix}
1 & 3 & 0 \\
0 & 2 & 1 \\
0 & 1 & 1
\end{pmatrix}
\]
is diagonalizable with minimum calculation \( \textbf{(10)} \)

Q.4. (a) Every group is isomorphic to permutation group \( \textbf{(7)} \)

(b) Show that for \( n \geq 3 \) \( Z (S_n) = 1 \) \( \textbf{(6)} \)

(c) Let \( A, B \) be two ideal of a ring, then \( \frac{A+B}{A} = \frac{B}{A \cap B} \). \( \textbf{(7)} \)

Q.5. (a) Verify Cayley – Hamilton theorem for the matrix \( \textbf{(7)} \)

\[
A = \begin{pmatrix}
0 & 1 & 2 \\
2 & -3 & 0 \\
1 & 1 & 1
\end{pmatrix}
\]

(b) Prove that ring \( A = Z \), the set of all integers is a principal ideal ring. \( \textbf{(7)} \)

(c) Under what condition on the scalar, do the vectors \((1, 1, 1), (1, \xi, \xi^2), (1, -\xi, \xi^2)\) form basis of \( c^3 \)? \( \textbf{(6)} \)

SECTION – B

Q.6. (a) Show that T.N. = 0 for the helix \( \textbf{(10)} \)

\[
R(t) = (a \cos \omega t) \mathbf{i} + (a \sin \omega t) \mathbf{j} + (bt) \mathbf{k}
\]

(b) The vector equation of ellipse \( x(t) = (2 \cos t) \mathbf{i} + (3 \sin t) \mathbf{j} ; (0 \leq t \leq 2\pi) \)

Find the curvature of ellipse at the end points of major & minor axes. \( \textbf{(10)} \)

Q.7. (a) Discuss & sketch the surface \( x^2 + 4y^2 - 4x - 4z^2 \) \( \textbf{(12)} \)

(b) Show that an equation to the right circular cone with vertex at 0, axis oz & semi – vertical angle \( \alpha \) is \( x^2 + y^2 = z^2 \tan^2 \alpha \). \( \textbf{(8)} \)

Q.8. (a) Show that hyperboloids of one sheet and hyperbolic parabolas are ruled surface. \( \textbf{(6+6)} \)

(b) Find an equation of the plane which passes through the point \((3, 4, 5)\) has an \( x \) – intercept equal to -5 and is perpendicular to the plane \( 2x + 3y - z = 8 \). \( \textbf{(8)} \)

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Roll Number